8.7 Estimation and Sample Size Determination for Finite Populations

Estimating the Mean

In Section 7.6, you used the **finite population correction (fpc) factor** to reduce the standard error by a value equal to $\sqrt{(N - n)/(N - 1)}$. When developing confidence interval estimates for population parameters, you use the finite population correction factor when samples are selected without replacement from a finite population. If more than 5% of the population is sampled (i.e., $n/N > 0.05$), the finite population correction factor significantly reduces the width of the confidence interval. If less than 5% of the population is sampled, the finite population correction factor has little or no practical effect on the confidence interval width. Equation (8.12) defines the $(1 - \alpha) \times 100\%$ confidence interval estimate for the mean.

CONFIDENCE INTERVAL FOR A MEAN (σ UNKNOWN)
FOR A FINITE POPULATION

$$\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} \sqrt{\frac{N - n}{N - 1}} \qquad (8.12)$$

To illustrate the finite population correction factor, refer to the confidence interval estimate for the mean developed for Saxon Home Improvement Company in Section 8.2. Suppose that in this month there are 5,000 sales invoices. Using $\bar{X} = \$110.27, S = \$28.95, N = 5,000, n = 100, df = n - 1 = 99$, and, with 95% confidence, $t_{\alpha/2} = 1.9842$. From Equation (8.12),

$$110.27 \pm (1.9842) \frac{28.95}{\sqrt{100}} \sqrt{\frac{5,000 - 100}{5,000 - 1}}$$

$$= 110.27 \pm 5.744(0.99)$$

$$= 110.27 \pm 5.69$$

$$\$104.58 \leq \mu \leq \$115.96$$

In this case, because only 2% of the population was sampled, the finite population correction has a minimal effect on the width of the confidence interval. To examine the effect of the correction factor when the sample size is more than 5% of the population size, see Example 8.10.

EXAMPLE 8.10

Estimating the Mean Force for Insulators

In Example 8.3 on page 290, a sample of 30 insulators was selected. Suppose the company produced a population of 300 insulators. Construct a 95% confidence interval estimate of the population mean.

SOLUTION Using the finite population correction factor, with, $\bar{X} = 1,723.4, S = 89.55, N = 300, n = 30, df = n - 1 = 29$, and $t_{\alpha/2} = 2.0452$ (for 95% confidence):

$$\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} \sqrt{\frac{N - n}{N - 1}}$$

$$= 1,723.4 \pm (2.0452) \frac{89.55}{\sqrt{30}} \sqrt{\frac{300 - 30}{300 - 1}}$$

$$= 1,723.4 \pm 33.44(0.9503)$$

$$= 1,723.4 \pm 31.776$$

$$1,691.62 \leq \mu \leq 1,755.18$$

Because 10% of the population was sampled, the finite population correction factor has a moderate effect on the confidence interval estimate.

Estimating the Proportion

In sampling without replacement from a finite population, the $(1 - \alpha) \times 100\%$ confidence interval estimate of the proportion is defined in Equation (8.13).

> CONFIDENCE INTERVAL ESTIMATE FOR THE PROPORTION USING
> THE FINITE POPULATION CORRECTION FACTOR
>
> $$p \pm Z\sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}$$
>
> **(8.13)**

To illustrate the use of the finite population correction factor when developing a confidence interval estimate of the population proportion, consider again the estimate developed for Saxon Home Improvement Company in Section 8.3. For these data, $N = 5,000, n = 100$, $p = 10/100 = 0.10$, and, with 95% confidence, $Z_{\alpha/2} = 1.96$. Using Equation (8.13),

$$p \pm Z\sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}$$

$$= 0.10 \pm (1.96)\sqrt{\frac{(0.10)(0.90)}{100}}\sqrt{\frac{5,000-100}{5,000-1}}$$

$$= 0.10 \pm (1.96)(0.03)(0.99)$$

$$= 0.10 \pm 0.0582$$

$$0.0418 \leq \pi \leq 0.1582$$

In this case, because only 2% of the population was sampled, the finite population correction factor has virtually no effect on the confidence interval estimate.

Determining the Sample Size

Just as you used the finite population correction factor to develop confidence interval estimates, you can also use it to determine sample size when sampling without replacement. For example, in estimating the mean, the sampling error is

$$e = Z\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

and in estimating the proportion, the sampling error is

$$e = Z\sqrt{\frac{\pi(1-\pi)}{n}}\sqrt{\frac{N-n}{N-1}}$$

To determine the sample size in estimating the mean or the proportion from Equations (8.4) and (8.5),

$$n_0 = \frac{Z^2\sigma^2}{e^2} \quad \text{and} \quad n_0 = \frac{Z^2\pi(1-\pi)}{e^2}$$

where n_0 is the sample size, without considering the finite population correction factor. Applying the finite population correction factor results in the actual sample size, n, computed as in Equation (8.14).

> SAMPLE SIZE DETERMINATION USING THE FINITE POPULATION
> CORRECTION FACTOR
>
> $$n = \frac{n_0 N}{n_0 + (N-1)}$$
>
> **(8.14)**

In determining the sample size for Saxon Home Improvement Company, a sample size of 97 was needed (rounded up from 96.04) for the mean and a sample of 100 (rounded up from 99.96) was needed for the proportion. Using the finite population correction factor in Equation (8.14) for the mean, with $N = 5,000$, $e = \$5$, $S = \$25$, and $Z_{\alpha/2} = 1.96$ (for 95% confidence), leads to

$$n = \frac{(96.04)(5,000)}{96.04 + (5,000 - 1)} = 94.24$$

Thus, $n = 95$.

Using the finite population correction factor in Equation (8.14) for the proportion, with $N = 5,000$, $e = 0.07$, $\pi = 0.15$, and $Z_{\alpha/2} = 1.96$ (for 95% confidence),

$$n = \frac{(99.96)(5,000)}{99.96 + (5,000 - 1)} = 98.02$$

Thus, $n = 99$.

To satisfy *both requirements simultaneously with one sample*, the larger sample size of 99 is needed.

Problems for Section 8.7

LEARNING THE BASICS

8.89 If, $\overline{X} = 75$, $S = 24$, $n = 36$, and $N = 200$, construct a 95% confidence interval estimate of the population mean, μ, if sampling is done without replacement.

8.90 Consider a population of 1,000, where the standard deviation is assumed to be equal to 20. What sample size is required if sampling is done without replacement if you desire 95% confidence and a sampling error of ± 5?

APPLYING THE CONCEPTS

8.91 The quality control manager at a light bulb factory needs to estimate the mean life of a shipment of 2,000 light bulbs. Assume that the standard deviation is 100 hours and that sampling is done without replacement.
a. Construct a 95% confidence interval estimate of the population mean life of light bulbs in this shipment if a random sample of 50 light bulbs selected from the shipment indicates a sample mean life of 350 hours.
b. Determine the sample size needed to estimate the population mean life to within ± 20 hours with 95% confidence.
c. What are your answers to (a) and (b) if the shipment contains 1,000 light bulbs?

8.92 A survey is planned to determine the mean annual family medical expenses of the 3,000 employees of a large company. The management of the company wishes to be 95% confident that the sample mean is correct to within $\pm\$50$ of the mean annual family medical expenses. A small-scale study indicates that the standard deviation is approximately $400. How large a sample size is necessary if sampling is done without replacement?

8.93 The manager of a bank that has 1,000 depositors wants you to estimate the proportion of its depositors with more than one account at the bank. A random sample of 100 depositors is selected without replacement, and 30 state that they have more than one account at the bank.
a. Construct a 90% confidence interval estimate of the population proportion of the bank's depositors who have more than one account at the bank.
b. What sample size is needed to estimate the population proportion to within ± 0.05 with 90% confidence?
c. What are your answers to (a) and (b) if the bank has 2,000 depositors?

8.94 An automobile dealer wants to estimate the proportion of customers who still own the cars they purchased five years earlier. Sales records indicate that the population of owners is 4,000. A random sample of 200 customers selected without replacement from the automobile dealer's records indicate that 82 still own cars that were purchased five years earlier.
a. Construct a 95% confidence interval estimate of the population proportion of all customers who still own their cars five years after they were purchased.
b. What sample size is necessary to estimate the population proportion to within ± 0.025 with 95% confidence?
c. What are your answers to (a) and (b) if the population consists of 6,000 owners?

8.95 The inspection division of the Lee County Weights and Measures Department is interested in estimating the mean amount of soft drink that is placed in 2-liter bottles at the local bottling plant of a large nationally known soft-drink company. The population consists of 2,000 bottles.

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The bottling plant has informed the inspection division that the standard deviation for 2-liter bottles is 0.05 liter.

a. Construct a 95% confidence interval estimate of the population mean amount of soft drink per bottle if a random sample of 100 2-liter bottles selected without replacement indicates a sample mean of 1.99 liters.

b. Determine the sample size necessary to estimate the population mean amount to within ±0.01 liter with 95% confidence.

c. What are your answers to (a) and (b) if the population consists of 1,000 bottles?

8.96 A stationery store wants to estimate the mean retail value of the 300 greeting cards that it has in its inventory.

a. Construct a 95% confidence interval estimate of the population mean value of all greeting cards that are in its inventory if a random sample of 20 greeting cards selected without replacement indicates a mean of $2.55 and a standard deviation of $0.44.

b. What is your answer to (a) if the store has 500 greeting cards in its inventory?